SCIENTIFIC AMERICAN, volume 13, December, 1921: "Eliakim Hastings Moore," 137 ["Following the old rule that the office of President of the American Association for the Advancement of Science should pass from a representative of the natural sciences to one of the physical sciences, this year a prominent entomologist gives place to a distinguished mathematician. The high office of President of the Association is almost without exception conferred upon the foremost representative of some special branch of science in the United States and therefore Professor Moore's election at the Chicago meeting held last December at once indicates that in the opinion of his colleagues he ranks first among American mathematicians, a decision in which there can be no dissenting voice."]

ZEITSCHRIFT FÜR MATHEMATISCHEN UND NATURWISSENSCHAFTLICHEN UNTERRICHT, volume 52, nos. 9–10, published September 1, 1921: "Ein Schaubild zur Darstellung der Zeit-Raum-Verhältnisse in der speziellen Relativitätstheorie" by F. P. Liesegang, 193–201; "Zur Einführung des Integralbegriffes" by A. Harnack, 201–205; "Das Prinzip der vollständigen Induktion. Seine Geschichte und Anwendung im mathematischen Unterricht" by W. Lorey, 205–209; "Kleine Mitteilungen," 209–219; "Aufgabenrepertorium," 219–224; "Bücherbesprechungen," 235–245.

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL, OTTO DUNKEL, AND H. P. MANNING.

Send all communications about Problems and Solutions to B. F. FINKEL, Springfield, Mo.

PROBLEMS FOR SOLUTION.

[N. B. Problems containing results believed to be new, or extensions of old results, are especially sought. The editorial work would be greatly facilitated if, on sending in problems, proposers would also enclose any solutions or information that will assist the editors in checking the statements. In general, problems in well-known text-books, or results found in readily accessible sources, will not be proposed as problems for solution in the MONTHLY. In so far as possible, however, the editors will be glad to assist the members of the Association with their difficulties in the solution of such problems.]

2959. Proposed by J. W. M. WEDDERBURN, Princeton University.

Solve the functional equation, $[g(x)]^2 = -2x + g(x^2)$.

2960. Proposed by E. P. LANE, University of Wisconsin.

When do two cones circumscribing a sphere intersect in two ellipses and when are the planes of the ellipses perpendicular?

2961. Proposed by J. L. RILEY, Stephenville, Texas.

Being given a triangle ABC, to determine two points P, P' such that the angles PBC, PCA, PAB are equal; also that the angles $P'CB$, $P'AC$, $P'BA$ are equal. Find also the equation of the circle which passes through the points P, P' and through the center of the circle circumscribing the triangle ABC.

2962. Proposed by R. M. MATHEWS, Wesleyan University.

To construct a triangle similar to a given triangle with its vertices lying on: (*a*) any three coplanar lines; (*b*) any three lines in space. (See Problem 2895, *1921*, 184.)

2963. Proposed by NATHAN ALTSHILLER-COURT, University of Oklahoma.

Through a given point to draw a line so that the sum of the squares constructed on the two segments cut off by it on the sides of a given angle should be equivalent to a given square.

2964. Proposed by N. P. PANDYA, Amreli (Kathiaward), India.

$ABCD$ is a quadrilateral circumscribed about a circle and an ellipse. AB touches the ellipse at P and the circle at Q. DC touches the same ellipse and the same circle at L and M, respectively. AD touches the ellipse at K. BC touches the circle at E. Find the condition that $PEMK$ may be a parallelogram.

2965. Proposed by C. N. MILLS, Tiffin, Ohio.

If a quadrilateral inscribed in a square has the diagonals a and b, and the area A, show that the area of the square is $\dfrac{a^2b^2 - 4A^2}{a^2 + b^2 - 4A}$.

SOLUTIONS

2824 [1920, 185]. Proposed by G. Y. SOSNOW, Newark, N. J.

If n_1, n_2, n_3, n_4 be the lengths of the four normals and t_1, t_2, t_3 the lengths of the three tangents drawn from any point to the semi-cubical parabola, $ay^2 = x^3$, then will $27n_1n_2n_3n_4 = at_1t_2t_3$ [From *Mathematical Tripos Examination*, Cambridge, England].

SOLUTION BY J. B. REYNOLDS, Lehigh University.

Let the parametric equations of the curve be

$$x = au^2 \quad \text{and} \quad y = au^3.$$

Then

$$\frac{dy}{dx} = \frac{3u}{2},$$

the equations of the normal and tangent will be

$$3au^4 + 2au^2 - 3yu - 2x = 0 \tag{1}$$

and

$$au^3 - 3xu + 2y = 0, \tag{2}$$

and the lengths of the normal and tangent from (x, y) to the curve will be

$$n = \left(\frac{x - au^2}{3u}\right)\sqrt{9u^2 + 4} \quad \text{and} \quad t = \left(\frac{x - au^2}{2}\right)\sqrt{9u^2 + 4}.$$

The polynomial whose roots are the squares of the roots of (1) is easily found[1] to be:

$$9a^2z^4 + 12a^2z^3 + 4a(a - 3x)z^2 - (8ax + 9y^2)z + 4x^2 = 9a^2\Pi(z - u_i^2).$$

Multiply the roots by a, according to the familiar rule, and then replace z by x. This gives

$$\Pi(x - au_i^2) = x(x^3 - ay^2).$$

Also multiplying the roots by 9 and replacing z by -4 we obtain

$$\Pi(4 + 9u_i^2) = \frac{4}{a^2}[729(x^2 + y^2) + 216ax + 16a^2].$$

Hence

$$n_1n_2n_3n_4 = -\frac{x^3 - ay^2}{27}[729(x^2 + y^2) + 216ax + 16a^2]^{1/2}.$$

In a similar manner from equation (2) we get,

$$\Pi(x - au_i^2) = 4(x^3 - ay^2),$$

$$\Pi(4 + 9u_i^2) = \frac{4}{a^2}[729(x^2 + y^2) + 216ax + 16a^2];$$

and hence,

$$t_1t_2t_3 = \frac{x^3 - ay^2}{a}[729(x^2 + y^2) + 216ax + 16a^2]^{1/2}.$$

Therefore

$$at_1t_2t_3 = 27n_1n_2n_3n_4$$

neglecting the sign.

2834 [1920, 273]. Proposed by OTTO DUNKEL, Washington University.

In any triangle ABC let M and N be, respectively, the points in which the median and the bisector of the angle at A meet the side BC, Q and P the points in which the perpendicular at N to NA meets MA and BA, respectively, and O the point in which the perpendicular at P to

[1] See, for example, Todhunter, *An Elementary Treatise on the Theory of Equations*, London, 1880, p. 36; or Salmon, *Lessons Introductory to the Modern Higher Algebra*, Dublin, 1885, p. 350.—EDITORS.